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Article

Future Prediction of Universe through Quantum Entanglement Entropy

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Abstract: In recent years, there has been a growing interest in research suggesting that the spacetime of the universe is formed by quantum entanglement. This paper aims to predict the future of the universe based on this quantum entanglement. Quantum entanglement follows the area law that it is proportional to the area of the boundary surface between two regions. Assuming that this quantum entanglement fundamentally controls the volume of the universe through the Holographic Principle, known as the Ryu-Takayanagi formula, this paper explores the implications. Furthermore, it is hypothesized that the area law breaks at extremely low temperatures, where quantum entanglement becomes proportional to the inverse power of the temperature T, ranging from $T^{-1/5}$ to $T^{-2/3}$. Applying this to the quantum entanglement in the universe, it is assumed that the breakdown of the area law occurs at these extremely low temperatures. Therefore, as the universe expands and the temperature becomes extremely low, where the area law breaks, quantum entanglement decreases, and the expansion of the universe comes to a halt. At this moment, there is a possibility that an energy gap emerges. This paper discusses the cessation of the universe's expansion, leaving the exploration of what follows as a future research topic.

Keywords: universe, quantum entanglement, holographic principle, area law, Ryu-Takayanagi formula

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1. Introduction

It has become clear that the universe is undergoing accelerated expansion [1,2], and furthermore, it is evident that this expansion involves the expansion of the space of the universe itself. However, the cause of this expansion remains uncertain, with various theories yet to provide a definitive explanation [3]. Therefore, the question arises whether the expansion of the universe will continue indefinitely or there will be a point where the expansion ceases and the contraction begins, whose answer remains unclear. What lies ahead for the universe, and is there a method to predict it based on the latest research in physics? Considering that the expansion of the universe involves the expansion of space itself, one must contemplate whether spacetime is inherently pre-existing or if it is associated with the emergence of matter.

Recent advancements in the study of quantum entanglement have shed light on the nature of space and time [4-6]. Moreover, the perspective that spacetime emerges from quantum entanglement and the corresponding research on this topic have made significant progress [7-9]. This paper adopts this standpoint, asserting that the spacetime of the universe emerges from quantum entanglement, and the expansion of cosmic space is caused by quantum entanglement. Therefore, in order to predict the future of the universe based on this quantum entanglement, it is essential to first identify particles that induce quantum entanglement throughout the entire cosmic space.

Photons exist in the vast cosmic space, confirmed by the Cosmic Microwave Background (CMB) [10,11]. Neutrinos also exist in the expansive cosmic space, known as cosmic background neutrinos [12]. However, it has been revealed that neutrinos have mass [13], and it is unlikely that neutrinos, like photons, can naturally and readily undergo pair production. Therefore, in this paper, we assume that photons are the particles responsible for generating the majority of quantum entanglement throughout the entire cosmic space.

So, how can we consider the relationship between quantum entanglement entropy and dark energy? The observationally established accelerated expansion of the universe is believed to be described through Einstein's equations. Einstein's gravitational field equations solved the relationship between the geometric structure of spacetime and the distribution of matter. Efforts continue to clarify the origin of dark energy, which drives the expansion of the universe, using concepts such as the cosmological constant and quantum effects of matter (*e.g.*, [14]).

This paper adopts the stance that the volume of the universe is controlled by the quantum entanglement entropy, which is calculated using the Ryu-Takayanagi formula. In other words, the spacetime of the universe emerges from quantum entanglement, and phenomena such as the expansion of the universe are caused by quantum entanglement. There is a suggestion, through holography, that classical properties of gravity, such as the Einstein equations, emerge from quantum entanglement [15-17]. Based on this, it can be considered that quantum entanglement entropy either significantly contributes to dark energy or is dark energy itself in our universe.

The theoretical relationship between dark energy and quantum entanglement entropy involves the challenging issue of unifying quantum mechanics and gravitational field theory through the holographic principle. This remains an open problem, and we look forward to future developments in research. Furthermore, it would be useful to experimentally create environments similar to outer space, in addition to studying the actual cosmic expansion, in order to investigate changes in quantum entanglement entropy and the rate of expansion of such spaces [18].

2. Photons of Cosmic Microwave Background Radiation

In this section, we discuss photons of cosmic microwave background radiation. After examining the current temperature and photon number density from observational data of cosmic microwave background radiation, we predict the approximate time when it reached a temperature of 1 K.

In this paper, it is argued that the quantum entanglement of photons existing throughout the entire universe controls the volume of the universe through the Ryu-Takayanagi formula. There is a viewpoint that this quantum entanglement is proportional to the surface area of the boundary between two regions, but as the temperature approaches absolute zero, this surface law breaks down, and the quantum entanglement decreases.

The cause of the breakdown of the surface law has not yet been experimentally confirmed, but this paper hypothesizes that it is due to some kind of phase transition. However, the Bose-Einstein condensation (BEC) of photons has only been confirmed under special conditions, and the BEC of photons in outer space has not been observed, meaning the critical temperature is unknown. In this paper, it is assumed that the critical temperature of the photons in the universe, that is, the temperature at which the breakdown of the quantum entanglement surface law begins, occurs at a certain temperature below 1 K. Please refer to Hypothesis 2 in the Discussion.

First, let us consider the relationship between the size and time of the universe based on the Friedman cosmological model [19]. If we denote a(t) as the scale factor of the universe at time t, during the matter dominant era, it is given by

$$a(t) \propto t^{2/3} \tag{1}$$

and during the cosmological constant dominant era, it is given by

$$a(t) \propto exp\left(H_0\sqrt{\Omega_\Lambda}t\right) \tag{2}$$

where H_0 is the Hubble constant and Ω_Λ is the cosmological constant.

Taking the natural logarithm of both equations reveals that, with increasing t, the increase in the scale factor of the universe $a(t)$ is larger for the latter than the former. Consequently, the temperature of photons present in cosmic space decreases more rapidly when calculated using the latter formula compared to the former. In other words, to cool photons, the former requires more time than the latter. The precision and accuracy of the numerical value of the time required to cool photons do not significantly impact the essence of this paper. What matters here is understanding approximately how much time it takes for the photons of cosmic background radiation to decrease in temperature to 1 K. Therefore, a rough calculation is performed using a slightly modified version of the former equation.

In the subsequent analysis, the cosmic age at the transition from the radiation dominant era (the period when radiation and matter are in thermal equilibrium) to the matter dominant era is set to 45,000 years from the beginning of the universe. At that time, the temperature of radiation and matter is assumed to be 11,000 K. With these assumptions, the following equations are obtained [20]:

$$T_{rad}(t) = 11000\left(\frac{t}{45000}\right)^{-2/3} \tag{3}$$

$$T_{mat}(t) = 11000\left(\frac{t}{45000}\right)^{-4/3} \tag{4}$$

where the former is the formula for radiation, the latter is the formula for matter, T represents temperature, and t represents time since the creation of the universe. Therefore, the calculation of photons from the cosmic background radiation is expressed by Equation (3). Before that, the current temperature and number density of photons in the cosmic background radiation are estimated as 2.725 K and 411/cm^3 [20].

The temperature of photons described above can be roughly calculated from Equation (3). Once the temperature of photons is known, the number density of photons can be calculated. When Equation (3) is rearranged,

$$t = 11000^{3/2} \times 45000 \times T^{-3/2} \tag{5}$$

and assuming T as 1 K, the value of t becomes approximately 51.916 billion years.

Although this is an extremely rough calculation, it is believed that the photons of the cosmic microwave background radiation will reach 1 K approximately 52 billion years after the creation of the universe. As mentioned earlier, using the equation for the cosmological constant dominant era of the universe, it is suggested that this occurs even earlier than this calculated value.

3. Emergence of Spacetime from Quantum Entanglement

In this section, we prepare to delve into the main discussion. The inception of the connection between quantum entanglement and the spacetime of the universe can be traced back to the Beckenstein-Hawking formula, which states that the entropy of a black hole is proportional to the area of the black hole's event horizon [21, 22]:

$$S_{\mathrm{BH}} = \frac{A_{BH}}{4G_{\mathrm{N}}} \tag{6}$$

where S_{BH} is the entropy of black hole, G_{N} is the gravitational constant, and A_{BH} is the area of the black hole's event horizon. Gauge-gravity correspondence, and more specifically AdS/CFT correspondence, posits that in an anti-de Sitter spacetime, the quantum gravity theory is equivalent to a one lower dimensional conformal field theory on its boundary, as asserted in reference [23].

Furthermore, the Ryu-Takayanagi formula [24] provides a calculation of quantum entanglement entropy in the context of gauge-gravity duality, establishing a connection between quantum entanglement entropy and spacetime:

$$S_A = \frac{A(\Gamma_A)}{4G_{\mathrm{N}}} \tag{7}$$

where S_A represents the entanglement entropy when dividing the d-dimensional space into region A, defined on the (d+1)-dimensional spacetime surface of constant time in conformal field theory, and its complement (region B). $A(\Gamma_A)$ corresponds to the d-dimensional configuration in the (d+2)-dimensional anti-de Sitter spacetime, where $A(\Gamma_A)$ is the minimal surface (minimal configuration) on the boundary $\delta\Gamma_A$ that matches the boundary δA of A with the smallest area.

Related research continues to evolve, gradually elucidating the mechanism by which the spacetime underlying the gravity of general relativity is born from quantum entanglement [25]. One of the challenges pointed out in the AdS/CFT correspondence is that the spacetime of the actual universe is described not by anti-de Sitter spacetime but by de Sitter spacetime. Regarding this, a holographic theory for classical gravity on a 3-dimensional de Sitter space has been proposed [26]. Furthermore, the previously mentioned Ryu-Takayanagi formula [24], when rewritten using the Planck length, can be expressed as [23]:

$$S_A = \frac{A(\Gamma_A)}{4{l_p}^d} \tag{8}$$

where ${l_p}^d$ is the Planck length in d+2 dimensions, and the quantum entanglement entropy is equal to the minimal area of spacetime in gravitational theory.

Based on the above, this paper takes the position that the spacetime of the universe emerges from the quantum entanglement of a quantum many-body system. In other words, it is assumed that spacetime did not inherently exist but emerged from matter. Furthermore, the cause of the expansion (including contraction) of the cosmic space is assumed to be related to quantum entanglement entropy.

4. Quantum Entanglement in the Universe at Cryogenic Temperatures and Predicting the Future of the Universe

In this section, we contemplate the methods for measuring the quantum entanglement entropy of the universe, as well as its quantification. Following that, we formulate the breakdown of the area law for quantum entanglement at extremely low temperatures. In formulating the violation of the entanglement area law at extremely low temperatures, it is concluded that quantum entanglement entropy in the universe suddenly decreases. Consequently, cosmic expansion halts, leading to significant changes in the universe.

4.1. Measurement of the quantum entanglement entropy of the universe

As stated in Introduction, we assume that throughout the vast expanse of cosmic space, the only entities present are photons of the cosmic microwave background radiation, and they are entangled in quantum states. Furthermore, we assume that these photons, while not in thermal equilibrium due to the expansion of the universe, exhibit a gradual temperature change when the temperature drops below 1 K. We also assume that at temperatures below 1 K, these photons can be approximated by the physical quantities of thermal equilibrium, allowing us to quantify entanglement entropy.

Furthermore, we assume that the universe is a sphere without holes. The questions is how to measure the quantum entanglement entropy of the universe. Let us assume that the Hubble parameter at the present time (*i.e.*, the Hubble constant) is 72 kms^{-1}Mpc^{-1} [27], the age of the universe is 13.8 billion years [28], and the radius of the observable universe is 46.5 billion light years [29].

First, let us roughly calculate the observable universe's radius 50 billion years after the creation of the universe. Assuming that the expansion continues at the same rate as it has from the beginning until now, during that period, calculating (50.0 / 13.8) × 46.5, we get approximately 168.5 billion light years. Considering the fact that the universe is expanding at an accelerating rate, the actual value is thought to be larger than this.

Second, assuming that even at 50 billion years after the creation of the universe, the Hubble constant is 72 kms^{-1}Mpc^{-1}, and we can roughly calculate the distance at which the expansion velocity reaches the speed of light at that time as 300,000 × 326 / 72, we get about 13.6 billion light years, where the number 326 is 1 Mpc expressed in light years [27]. The unit is 10,000. Considering the fact that the universe is expanding at an accelerating rate, the actual value is likely to be smaller than this value of 13.6 billion light years. Therefore, the expansion rate near the outermost surface of the observable universe is far above the speed of light.

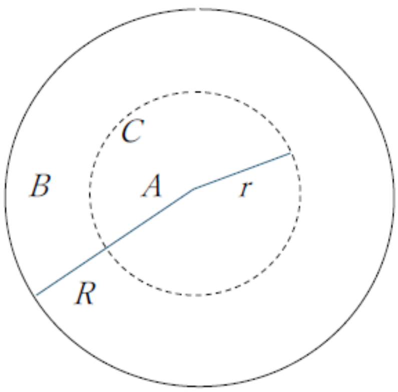

Figure 1. Diagram of the entire universe at 50 billion years after the cosmic creation, where the solid line represents the outermost surface of the actual universe, and the dashed line represents the outermost surface of the observable universe.

Figure 1 represents the entire universe after 50 billion years of cosmic evolution. The universe is divided into two regions. A is the region inside the outermost surface of the observable universe, indicated by the dashed line. As described above, the expansion rate of the part at the surface of sphere A is equal to or greater than the speed of light. B is the region outside sphere A and inside the outermost surface of the actual expanse of the universe. An observer is located at point C, just inside the dashed line. Since the expansion velocity of the sphere represented by the dashed line is greater than the speed of light, the observer at point C cannot observe the internal state of region B. The unobservable information of B is given by entanglement entropy. Denoting this entropy as S_B and the information content of A as S_A, since S_B=S_A, it can be quantified using the Ryu-Takayanagi formula [24]. If we let R be the radius of the actual spatial expansion of the universe, and r be the radius of sphere A, the distance from the center of the sphere to the observer can also be considered as r.

It is important to note that the observer's position is influenced by the theory of relativity. In other words, the metrics of the surface of sphere A are determined by the mass and energy inside sphere A. The quantity is not formulated, so the coordinates of the observer's position are also undetermined, and the physical quantities observed from the observer's perspective also become indeterminate. Despite this issue, in this paper, we proceed with the discussion under the assumption that physical quantities, including the position of the observer, can be approximated using classical mechanics.

$$R(t) = Kr(t)\ (K > 1) \tag{9}$$

$$H(t) = \frac{\dot{a}(t)}{a(t)} \tag{10}$$

$$S_B = S_A = F4\pi r(t)^2 = F4\pi K^{-2} R(t)^2 \tag{11}$$

$$R(t) = Qa(t) = QH(t)^{-1}\dot{a}(t) \tag{12}$$

Q is considered to be the proportionality constant between $R(t)$ and $a(t)$. Additionally, K is a constant within the range in the parentheses of Equation (9). Strictly speaking, it is considered to be a variable greater than 1 that changes with time. However, for the discussion of this paper, the exactitude of the value of K is not so important, and its variation with time is estimated as extremely small. Therefore, it is regarded as a constant for ease of calculation here. Furthermore, F is the proportionality constant for the boundary surface area of $S_B = S_A$, which is the surface area of the boundary between region A and region B. This expression arises from the rule known as the "area law" for quantum entanglement entropy [30]. In other words, the area law is assumed to be applicable in cosmic space. H represents the Hubble parameter, considered as a variable, changing with time. Also, $a(t)$ represents the scale factor of the universe, and $\dot{a}(t)$ denotes its time derivative.

To obtain approximate calculation formulas and values, the following conditions are imposed:

1) The value of the Hubble parameter does not change for 100million years.
2) The Hubble parameter increases by the same value, every 100 million years until the onset of the breaking of the area law of the quantum entanglement entropy of photons of the cosmic background radiation.
3) The same value for R , the distance from the center of the sphere to the outmost boundary of the observable universe , is used for each 100 million year period.

Again, the above three conditions are limited to a period in which the Hubble parameter has positive values, that is, a period when the expansion of the universe is occurring. It should be noted that these hypothetical condition are not scientifically precise, but are intended to give an overview of the cosmic picture after tens of billions years, and are models for that purpose. However, if we want to make the equation more precise, it is possible to decrease the length of the 100 million year units in the equation to 10,000 years for example. In that case, N in the equation would become larger. Here, we adopt the unit of 100 million years. Then, the following equation is obtained.

$$\begin{aligned} r_{N+1} = {} & 465\ [10^8 \text{ light years}]\ (1 + 72\ [\text{km}s^{-1}\text{Mp}c^{-1}] \times 1\ [10^8 \text{ million years}]) \\ & \times \{1 + (72 + h) \times 1\} \cdots \{1 + (72 + Nh) \times 1\} \end{aligned} \tag{13}$$

where h represents the amount by which the Hubble constant increases every 100 million years.

Generalizing the above equation , we obtain the following equation:

$$\begin{aligned} r_{N+1} &= r_0(1 + H_0 \times 1)\{1 + (H_0 + h) \times 1\} \cdots \{1 + (H_0 + Nh) \times 1\} \\ &= r_0 \prod_{n=1}^{N} (1 + H_0 + nh) \end{aligned} \tag{14}$$

where r_0 represents the current radius of the observable universe, and H_0 represents the current Hubble constant. Equations (13) and (14) above are expressions for r in (138 + 1×N) hundred million years, all (72 + nh), (H_0 + Nh) are followed by a unit of $\text{kms}^{-1}\text{Mpc}^{-1}$, and all the number 1 are followed by a unit of 100 million. N is a positive integer. In addition, in order to obtain the exact number of r_{N+1}, it is necessary to unify all units of 100 million light years, Mpc^{-1}, and 100 million years. However, this is not necessary because the purpose of this section is to look at the picture of the universe. It can be easily calculated from Equation (9), as $R(t)$ can be obtained by multiplying $r(t)$ by K.

4.2. Breaking the area law of quantum entanglement at cryogenic temperatures

As a rough calculation, photons in the cosmic background radiation reach 1 K at about 52 billion years after the creation of the universe. Below that temperature, we assume that the quantum entanglement entropy of photons can be quantified and expressed by the Ryu-Takayanagi formula [24] as described above.

Generally, the size of quantum entanglement follows the area law. That is, the magnitude of quantum entanglement between regions is proportional to the size (area) of the boundary of the region at room temperature. However, at low temperatures, as the temperature T approaches absolute zero, the maximum value breaks in proportion to the inverse of the temperature T to the 2/3 power [31]. However, what must be considered here is that the area law is not suddenly broken in proportion to the inverse of T to the 2/3 power as the temperature becomes lower, but rather it begins to break in proportion to the 1/5 power of the inverse of T, and the maximum value is considered to be the inverse of T to the 2/3 power [31]. Let us examine this in more depth.

Assuming that the entire photon system is in a thermal equilibrium state at low temperatures, the quantum entanglement entropy of the photons is assumed to be quantifiable. Photons are Bose particles. In quantum statistical mechanics, the Bose distribution function is as follows:

$$n(\varepsilon) = \frac{1}{e^{\beta(\varepsilon-\mu)}-1} \tag{15}$$

Here, ε represents the energy of a single particle state, μ is the chemical potential, β is equal to $1/kT$ where k is the Boltzmann constant, and $n(\varepsilon)$ denotes the average occupancy of a single-particle state with energy ε in the particle ensemble. Bose-Einstein condensation occurs in Bose particles at extremely low temperatures, leading to the following equation:

$$N_0(T) = N\left[1 - \frac{T}{T_c}^{3/2}\right] \tag{16}$$

where $N_0(T)$ is the number of Bose particles in the quantum state with $\varepsilon = 0$ at temperature T. N is the total number of Bose particles, and T_c is the critical temperature at which Bose particle condensation begins, that is, the temperature at which phase transition starts. When plotted on a graph, $N_0(T)$ looks as in Figure 2.

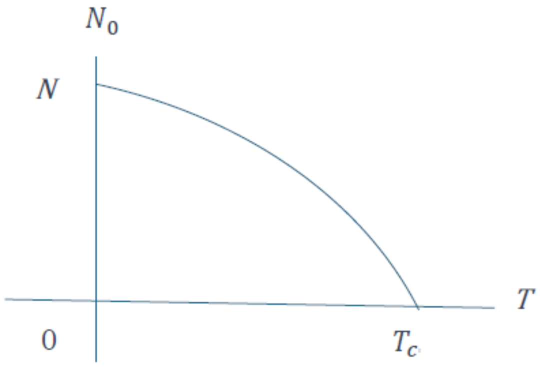

Figure 2. Temperature dependence of the number of condensed particles N_0, where T_c represents the critical temperature.

The curve in Figure 2 is continuous until the temperature reaches zero degrees from T_c. Photons are Bose particles, and their quantum entanglement entropy begins to violate the area law as the temperature becomes extremely low. Whether this is directly related to the condensation of Bose particles is currently uncertain.

However, it is possible for Bose-Einstein condensation (BEC) of photons to occur in a special environment where photons are confined using optical resonators or microstructures [32]. The critical temperature in this case is below 3.3 K. However, this is the critical temperature only for this experiment. This occurs under very specific conditions, and there is no evidence that it happens generally. Furthermore, there is currently no evidence linking BEC to a violation of the area law. Nonetheless, this is a change related to the violation of the area law at extremely low temperatures, and since the number of entangled photons decreases, we cannot completely rule out the possibility of a phase transition in photons.

In this paper, we hypothesize that the violation of the area law of photon entanglement at extremely low temperatures is somehow related to a phase transition. Although the exact critical temperature cannot be determined. Therefore, this paper assumes that the breakdown of the surface law begins once the temperature falls below 1 K. In a sense, it is conjectured that both quantum entanglement and entropy undergo a kind of phase transition at extremely low temperatures, where the area law begins to break down in proportion to the inverse fifth root of the temperature T. At that moment, the curve of quantum entanglement entropy becomes discontinuous.

4.3. Quantum entanglement entropy of the universe at cryogenic temperatures and future prediction of the universe

Here is something that must be kept in mind. The area law, which states that the entropy of quantum entanglement is proportional to the area of the boundary, has been thought to be due to the overwhelming likelihood of quantum entanglement between quantities that are very close to the boundary surface. However, it has been proven that this quantum entanglement occurs regardless of the distance from the boundary surface [30]. As mentioned earlier, research is also advancing on the emergence of spacetime from quantum entanglement. This paper adopts a similar standpoint. Namely, it assumes that the cause of the expansion or, in some case, contraction of cosmic space is attributed to quantum entanglement.

We now consider the expression for quantum entanglement entropy $S_A(t)$ per time periods. We let
t_1 be the time when the area law of quantum entanglement of a photon is proportional to the inverse of T the 1/5 power and begins to break,
t_2 be the time at which the photon's breaking of the area law of quantum entanglement becomes proportional to the inverse of T to the 2/3 power,
T_1 be the temperature of the photon at time t_1, and
T_2 be the temperature of the photon at time t_2.

a) $t < t_1$

$$S_A(t) = F\Gamma_A(t) \tag{17}$$

where the quantum entanglement entropy is proportional to the surface area of the boundary between region A and region B, and F is its proportionality constant. Note that $\Gamma_A(t)$ is not from the Ryu-Takayanagi formula but is the surface area of the sphere in region A. However, this $S_A(t)$ can be quantified by the Ryu-Takayanagi formula, as mentioned above. Investigating further at this point, Equations (17) and (11) are fundamentally the same. When Equation (11) is differentiated with respect to time t, the following expression is obtained:

$$\dot{S}_A(t) = 8F\pi K^{-2} r(t)\dot{r}(t) \tag{18}$$

Examining Equation (18), it can be observed that if $\dot{S}_A(t)$ becomes negative, also $\dot{R}(t)$ becomes negative. As indicated in b), this presents a significant problem.

b) $t_1 \leq t < t_2$

$$S_A(t) = F\Gamma_A(t)\left(1 - CT(t)^{-x(t)}\right) \tag{19}$$

where $1/5 \leq x(t) < 2/3$ [31], C is the proportionality constant that breaks the area law in proportion to the inverse of the temperature T and is assumed to be constant regardless of temperature, and $\Gamma_A(t)$ is just before the area law is broken. For the calculation of Equation (19), it is assumed that the physical meaning of the breaking of the area law is that the quantum entanglement entropy decreases by the amount of the broken area law.

Let us take a closer look at the interval b), especially around time t_1. At $t = t_1 - \Delta\varepsilon$, Equation (17) applies, and by letting $\Delta\varepsilon \to 0$, the amount of quantum entanglement entropy at this time $S'_A(t_1)$ becomes

$$S'_A(t_1) = F\Gamma_A(t_1) \tag{20}$$

Next, at $t = t_1 + \Delta\varepsilon$, Equation (19) applies, and by letting $\Delta\varepsilon \to 0$, the quantum entanglement entropy at this time $S"_A(t_1)$ becomes

$$S"_A(t_1) = F\Gamma_A(t_1)\left(1 - CT(t_1)^{\frac{-1}{5}}\right) \tag{21}$$

Since it can be estimated that $0 < 1 - CT(t)^{\frac{-1}{5}} < 1$, $S'_A(t)$ is greater than $S"_A(t)$. In other words, at time t_1, $S_A(t)$ abruptly decreases. Therefore, $\Gamma_A(t)$ also decreases. This means that the expansion of the universe comes to a halt. This occurs because $\dot{S}_A(t)$ becomes negative, causing $\dot{r}(t)$ and $\dot{R}(t)$ to become also negative. At time

t_1 when the temperature of photons becomes T_1, the expanding universe may suddenly begin to contract, leading to the emergence of an energy gap. However, this issue is not addressed in this paper.

Additionally, let us examine the moment immediately after time t_1. At time t_1, the temperature of photons is T_1 which is the temperature at which the area law of entanglement entropy of photons is violated. In other words, the entanglement entropy $S_A(t)$ of photons, which was increasing until just before reaching that temperature, decreases the moment the temperature drops to that level. As a result, the expansion of the universe halts, leading to a significant transformation in the cosmos.

Let us examine this with Equation (13) for the period in which the Hubble parameter functions as a positive value, that is, in the range of $t < t_1$. According to Equation (9), the radius of region A is R_{N+1} multiplied by K. Omitting units and $N+1$, we have

$$R = 465 \times (1 + 72 \times 1)\{1 + (72 + h) \times 1\} \cdots \{1 + (72 + Nh) \times 1\} \tag{22}$$

Now, let us differentiate Equation (22) with respect to t. During this process, all the terms before the last term $[1 + \{72 + (N-1)h\} \times 1]$ have already completed expansion and become constants. To simplify the calculation, let us designate the product of all these constants as 1.

Furthermore, considering that $72 + Nh$ represents the Hubble parameter, we denote it as $H_N(t)$, and we can eliminate the constant part. This leads to the following equation:

$$\dot{R} \propto H_N(t) = 72 + Nh \tag{23}$$

This is true in the range of $\boldsymbol{t < t_1}$ as described above. However, the moment t reaches $\boldsymbol{t_1}$, this equation no longer holds, and $\dot{\boldsymbol{r}}$ becomes negative. Then, $\dot{\boldsymbol{R}}$ also becomes negative because R is proportional to r according to Equation (9). Therefore, it is expected that the expansion of the universe will stop and an energy gap will arise due to a kind of phase transition**.** The energy gap in a phase transition refers to the difference in energy between different phases of a substance (e.g., solid, liquid, gas). In a first-order phase transition (e.g., the transition of water into ice), the energy of the substance changes abruptly before and after the transition. In this case, the energy can be expressed as a linear function, and the energy gap becomes clearly visible. In a second-order phase transition, the energy is expressed as a higher-order function, and the energy changes continuously, with no observable energy gap.

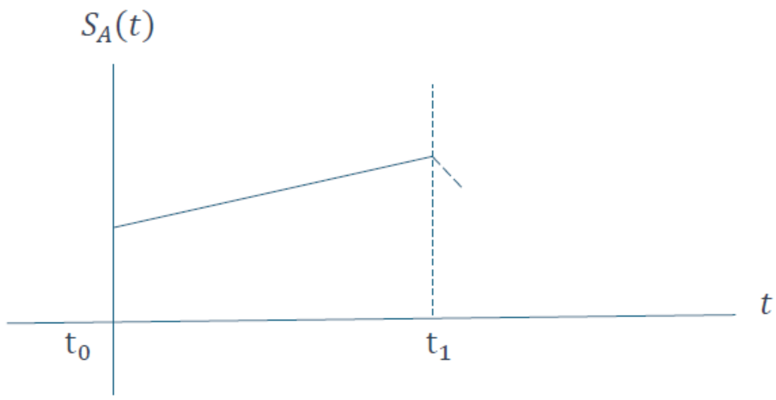

Figure 3. Relationship between quantum entanglement entropy of the universe and the age of the universe.

This paper proposes that violation of the area law in quantum entanglement at ultra-low temperatures could be considered a type of phase transition, and as a result, an energy gap may emerge. However, there is not yet sufficient evidence to support these ideas, and the mathematical framework has not been fully established, so these will be considered as future research topics. The next challenge is to solve this problem, and in this paper we conclude that the expansion of the universe stops.

In summary, the area law of entanglement entropy for photons is violated at the critical temperature $\boldsymbol{T_1}$, where the universe suddenly ceases to expand. Naturally, the conclusion takes into account the interdependence of both surface area and volume for regions A and the observable universe as indicated by Equation (9). Let us illustrate this with a diagram in Figure 3.

5. Discussion

In this paper, there are as many as 14 assumptions listed below:

1. Only photons exist throughout space and cause quantum entanglement throughout space.
2. At a certain low temperature below 1K, photons in space break the area law of their quantum entanglement entropy.
3. The breaking of that area law starts at 1/5 power of the inverse of the temperature, and at much lower temperatures, the maximum power reaches 2/3.
4. The spacetime of the universe emerges from the quantum entanglement of quantum many-body systems, and the expansion or contraction of space is caused by the quantum entanglement of photons in the universe.
5. The amount of quantum entanglement entropy of photons in space can be approximated by physical quantities in thermal equilibrium at low temperatures below 1K.
6. Quantum entanglement entropy of photons throughout the universe can also be quantified by the Ryu-Takayanagi formula.
7. The area law also applies to quantum entanglement in space, and its proportionality constant is constant regardless of temperature.
8. The proportionality constant for the breaking of the area law of photons in space does not vary with time.
9. Even in space, the amount of quantum entanglement entropy in the universe as a whole increases with time.
10. The universe is a sphere without holes.
11. Hubble's law can be applied regardless of the age of the universe.
12. Breaking of the area law of quantum entanglement at low temperatures is due to a phase transition.
13. The quantum entanglement entropy decreases by the amount of breakage of the area law of quantum entanglement at low temperatures.
14. The position of an observer in space can be approximated by classical mechanics.

The study of quantum entanglement will continue to progress, and the observation of the universe will also advance greatly in the future. We must continue to watch for results that will force us to change the above assumptions.

6. Conclusions

The universe does not necessarily continue to expand indefinitely for eternity. Based on the Ryu-Takayanagi formula, it is hypothesized that the size of the universe can be measured through the quantum entanglement entropy of space. Furthermore, it is suggested that this quantum entanglement entropy breaks the area law at extremely low temperatures, leading to a decrease in quantum entanglement entropy, which in turn causes the acceleration of the universe's expansion to cease. This is the main conclusion of this paper.

So, what will happen after the accelerated expansion of the universe comes to a halt? To understand this, we must first investigate the cause of the breakdown of the area law of the photon's quantum entanglement in space at extremely low temperatures. In this paper, it is assumed to be a type of phase transition. We need to examine whether this assumption is correct, and if it is, we must determine the critical temperature and derive the energy equation for this transition. Based on this, we must decide whether it is a first-order or second-order phase transition. If it is a first-order transition, the energy gap will be obtained.

If the assumption of a phase transition is incorrect, we must start by exploring why the area law breakdown occurs. Additionally, even if it is a phase transition, if it turns out to be a second-order phase transition, there will be no energy gap. In this case, we will need to study the consequences of the cessation of the universe's accelerated expansion from a different perspective. In conclusion, significant research challenges remain, and we will continue to work on these in the future.

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