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Article

Electron Model as a Spherical Standing Wave: Validation by Constant Calculation

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Abstract: Positivity, negativity, and wave-particle duality are fundamental properties in existence. Assuming the existence of a complex point charge with these properties, a spherical standing wave model is derived. This spherical standing wave is a spherically symmetric standing wave that resonates between the inner and outer radii. The inner radius does not become zero; therefore, no self-energy divergence problem exists. Since it has complex amplitude corresponding to voltage (scalar potential) and current (vector potential), it is compatible with quantum theory that also has complex amplitude. An electron model is assumed to be a spherical standing wave with an electron classical radius as the inner radius, a Bohr radius as the outer radius, and an elementary charge as the size of wave source. This study obtains energy and resonance conditions from an equivalent resonance circuit. The derived formulas include the Compton wavelength, electron mass, ionization energy, and Rydberg constant. Thus, the calculated values were consistent with existing physical constants.

Keywords: spherical wave, complex amplitude, Bohr radius, electron classical radius, fine structure constant, Compton wavelength, electron mass, ionization energy, principal quantum number, Rydberg constant

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1. Introduction

A sphere is among the most fundamental forms found in nature. Therefore, spherical waves are more appropriate than plane waves when considering waves inherent in nature or those that resonate with it. Electrons are not only representative of point charges but they also exist with waves. Electrons as point charges suffer from the self-energy divergence problem at the center [1]. Milo Wolff proposed a spherical standing wave with the form $e^{i\omega t}\sin kr/r$ as an electron model by subtracting the outward spherical wave from the inward spherical wave. The amplitude becomes finite when $r = 0$ [2-4].

The spherical standing wave proposed herein is a spherical wave (outward or inward) with a complex amplitude. It has two types of standing waves inside, namely, pressure and flow [5]. The inner and outer radii are determined by the resonance condition, and an important feature here is that waves exist only between them. The inner and outer radii, frequency (wave number), and energy have a close relationship defining other parameters with respect to each other [6]. The inner radius does not become zero; therefore, no self-energy divergence problem exists.

So far, quantum theory has developed at the expense of meaning and representation. However, searching for the possibility of describing elementary particles such as electrons using macroscopic structures is important because this world might be designed so that humankind could easily understand even at the level of quantum mechanics. For example, Shuichi Iida's VR electron model was such an attempt, where the electron was modeled as having a ring-like structure [7]. In this paper, a spherical standing wave is postulated as the electron model, and we consider its consistency with various parameters derived from conventional quantum mechanics. This paper was revised as a technical report of a previous study [8].

2. Fundamental wave of the spherical standing waves

The scalar field $\phi(r,t)$, wherein the distance from the center is r and the time variable is t, can express the scalar potential ϕ of a spherical wave as follows:

$$\phi(r,t) = \frac{q_0 e^{i(\omega t \mp kr)}}{4\pi\varepsilon_0 r} \tag{1}$$

where i is the imaginary unit; e is the base of the natural logarithm; ε_0 is the vacuum permittivity; ω is the angular frequency representing the temporal phase; and k is the wave number representing the spatial phase. The spherical wave amplitude is inversely proportional to the distance r from the center. The equivalent of the proportionality constant is the amplitude coefficient q_0, i.e., q_0 is a constant representing the size of the spherical wave source.

However, the scalar field in Eq. (1) is also the sum of the phase potential added to the electrostatic potential created by charge q_0. In the phase term $e^{i(\omega t - kr)}$, the iso-phase surface moves along the direction in which the radius r increases with time t. Therefore, it represents an outwardly directed spherical wave. Similarly, the phase term $e^{i(\omega t + kr)}$ represents an inwardly directed spherical wave and may also be regarded as the delayed (or advanced) potential of charge variation with respect to time $q = q_0 e^{i\omega t}$.

The angular frequency ω representing the temporal phase and the wave number k representing the spatial phase are related by $\omega = kc$. The constant c is a constant representing the speed at which the wave propagates. In the case of electromagnetic waves, it is generally a constant representing the speed of light.

According to the exact solution of the Maxwell equation for a spherical wave, the current and displacement current in a spherically symmetric spherical wave are in opposite directions with respect to one another and no magnetic field is generated [9]. Moreover, from a wave perspective, a spherical wave with a complex amplitude can be identically deformed as shown in Eq. (2). The inner portion of the wave shows that it comprises a voltage standing wave and a standing wave corresponding to the current [5].

$$\phi(r,t) = \frac{q_0 e^{i\omega t}}{4\pi\varepsilon_0 r}\cos kr + \frac{q_0 e^{i(\omega t \mp \pi/2)}}{4\pi\varepsilon_0 r}\sin kr \tag{2}$$

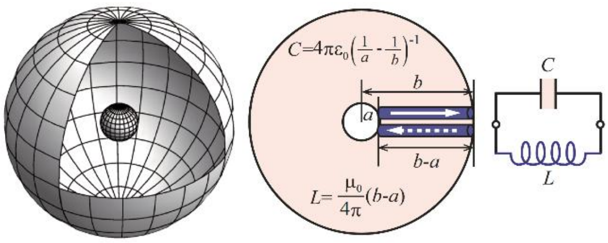

Figure 1. A circuit equivalent to a concentric spherical cavity resonator. *a*: inner radius; *b*: outer radius; *C*: capacitance of spherical capacitor; *L*: inductance of linear coil.

2.1. Energy and resonance conditions of the spherical standing wave

Consider a concentric spherical cavity surrounded by radii a and b ($a < b$) (Fig. 1). The energy density is expressed as follows [6]:

$$\frac{\varepsilon_0}{2}(\nabla\phi)\cdot(\nabla\phi^*) = \frac{q_0^2}{32\pi^2\varepsilon_0}\left(\frac{1}{r^4} + \frac{k^2}{r^2}\right) \tag{3}$$

where ϕ^* is the complex conjugate of ϕ. The total energy U is obtained by computing the definite integral on the spherical shell $4\pi r^2 dr$ of the section in which the wave exists [a, b].

$$U = \int_a^b \frac{\varepsilon_0}{2}(\nabla\phi)\cdot(\nabla\phi^*)4\pi r^2\,dr = \frac{q_0^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{q_0^2}{8\pi\varepsilon_0}k^2(b-a) \tag{4}$$

We obtained the following using the $1/c^2 = \varepsilon_0\mu_0$ relation:

$$U = \frac{q_0^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{b}\right) + \frac{\mu_0}{8\pi} q_0^2 k^2 c^2 (b - a) \tag{5}$$

The first term on the right-hand side of Eq. (5) is equivalent to the electrical energy $q_0^2/2C$ when the charge q_0 is stored in a capacitor of capacitance $C = 4\pi\varepsilon_0(1/a - 1/b)^{-1}$. The second term on the right side of Eq. (5) is equivalent to the current energy $L(\omega q_0)^2/2$ when a current $\omega q_0 (= kcq_0)$ flows through a coil with inductance $L = (\mu_0/4\pi)(a - b)$. In a resonance state, if both energies are observed to be equal, then

$$k^2 ab = 1 \tag{6}$$

and the resonance conditions are obtained [6].

3. Application to electron model

Consider the spherical standing wave as an electron model. First, let the magnitude of the spherical standing wave source be the elementary charge q_e, inner radius a be the electronic classical radius r_e, and outer radius b be the Bohr radius a_0.

$$r_e = \frac{q_e^2}{4\pi\varepsilon_0 m_e c^2} \Rightarrow a, \qquad a_0 = \frac{4\pi\varepsilon_0 \hbar^2}{m_e q_e^2} \Rightarrow b \tag{7}$$

Moreover, the fine structure constant [10] is expressed as $\alpha = q_e^2/4\pi\varepsilon_0\hbar c$, Henceforth, the expression

$$a = \alpha^2 b \tag{8}$$

will be used for the relationship obtained. In this paper, the inner radius a and outer radius b represent the classical electron radius and Bohr radius, respectively.

From Eqs. (6) and (8), the wave number k is obtained as

$$k = \frac{1}{\sqrt{ab}} = \frac{1}{\alpha b} \tag{9}$$

When the wavelength corresponding to this wave number k is calculated, it matches the Compton wavelength of the electron.

3.1 Total energy and mass of the electron

From Eqs. (5) and (8), the total electron energy U is given as

$$U = \frac{q_e^2}{8\pi\varepsilon_0} \cdot \frac{1}{a}(1 - \alpha^2) + \frac{\mu_0}{8\pi} q_e^2 k^2 c^2 b (1 - \alpha^2) \tag{10}$$

When the resonance condition $k^2 ab = 1$ is satisfied, the electrical and current energies become equal and may be expressed as

$$U = \frac{\mu_0}{4\pi} q_e^2 k^2 c^2 b (1 - \alpha^2) \tag{11}$$

From the equivalence of energy and mass, mass m is expressed as

$$m = \frac{U}{c^2} = \frac{\mu_0}{4\pi} q_e^2 k^2 b (1 - \alpha^2) \tag{12}$$

When this mass is calculated, it matches the existing electron mass within the range of four significant figures.

Although they are essentially in agreement, the electron model value is slightly smaller than the existing electron mass by approximately 0.005%. The fine structure constant is usually called the electromagnetic interaction coupling constant. Assuming that the term proportional to α^2 is the energy required for the interaction, the mass, m', can be expressed as

$$m' = \frac{\mu_0}{4\pi} q_e^2 k^2 b \tag{13}$$

When the mass, m', is calculated, it matches the existing physical constant, electron mass [11].

This is probably because the existing electron mass is based not on the zero energy, but rather on the energy (mass) of the interactable state. The energy U' corresponding to mass m' is obtained by omitting the term proportional to α^2 in Eq. (10):

$$U' = \frac{q_e^2}{8\pi\varepsilon_0} \cdot \frac{1}{a} + \frac{\mu_0}{8\pi} q_e^2 k^2 c^2 b \tag{14}$$

3.2 Ground-state electronic model

In the energy calculation for the spherical standing wave in Eq. (4), we consider the case where the wave is only between the radii a and b. Figure 2 shows the model in that state.

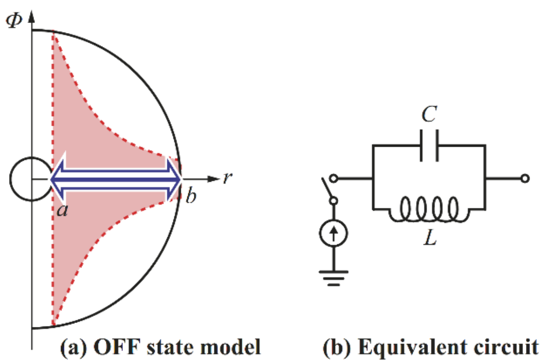

Figure 2. OFF-state electron model. The pale red (gray) area shows the voltage amplitude. The arrow on the horizontal axis depicts the current range (displacement current in the direction opposite to the current direction).

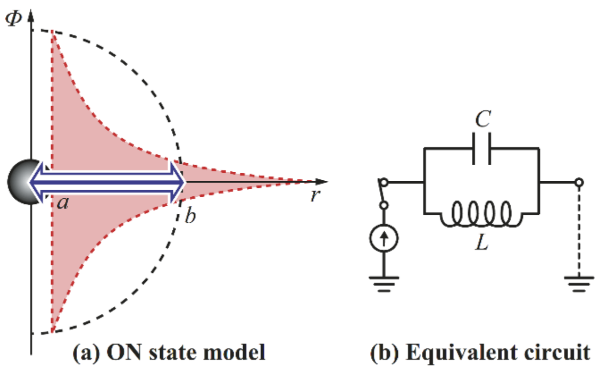

Figure 3. ON-state electron model. The switch of the current source is closed. The voltage amplitude exists from a to infinity. The current exists from 0 to b.

An LC circuit as an equivalent circuit, if it is connected to an external circuit, can be considered as an LC parallel circuit. In such an LC parallel circuit, the impedance seen from the outside takes the maximum value at the time of resonance. The existence of a wave source (current source) is implicitly assumed to be at the center of the spherical wave, but at resonance, the switch with that current source can be considered open. Thus, this state is called the OFF state and considered to be the so-called "zero energy state." It is the lowest in energy and represents the ground state of an electron.

3.3 Electron model of the ionic state

When considering the existence of electrons as a cause of phenomena, such as static electricity, the spatial range of the existence of the electrical field created by the charge of the electrons is thought to have spread to some extent.

Equation (5), which represents the energy of a spherical standing wave, is the sum of the electrical and current energies. Electrical energy greatly depends on the inner radius a, and the effect of the outer radius b is small. Conversely, the current energy greatly depends on the outer radius b, while the effect of the inner radius a is small. The energy U' in Eq. (14) is obtained by calculating the electrical energy up to $b = \infty$. The current energy corresponds to that calculated from $a = 0$.

Therefore, consider the case where an electric field, such as static electricity, is generated outside the outer radius b of the spherical standing wave (Fig. 3(a)). This is a case where a quasi-electrostatic wave has no current component, i.e., without movement of the equi-phase plane. However, the resonance condition does not change and has the same wave number k. Here, the same amount of current energy as the increased electric energy must be generated. This dual current energy is thought to broaden the range of existence from the inner radius a to the origin 0 (Fig. 3). Thus, the switch between the current source and the equivalent circuit is considered to be in the closed state, and the state is called the ON state.

In the OFF state, the outer radius b becomes a discontinuous surface of the electric field, and it appears as if a surface charge is generated there. Conversely, in the ON state, the outer radius b is no longer a discontinuous surface of the electric field and the surface charge generated in the OFF state appears as if it has moved to infinity. In chemistry, this ON state is considered to be an "ionized state," which is said to be a state wherein "+ ions and electrons are lost."

3.4 Total energy of the ionic state

The energy U' in Eq. (14) is thought to be the total energy of the ionic state. The energy density that generates this energy is expressed as

$$\frac{\varepsilon_0}{2}(\nabla\phi)\cdot(\nabla\phi^*) = \frac{q_e^2}{32\pi^2\varepsilon_0}\times\begin{cases} k^2/r^2 & [0,a] \\ \dfrac{1}{r^4}+\dfrac{k^2}{r^2} & [a,b] \\ 1/r^4 & [b,\infty] \end{cases} \tag{15}$$

However, there is not one potential candidate that can produce such an energy density. For example, in terms of the electric field generated outside the outer radius b, the electrostatic potential $q_e/4\pi\varepsilon_0 r$ generated by the charge q_e is one of the candidates from the viewpoint of energy only. From the wave continuity, the potential $q_e e^{i\omega t}/4\pi\varepsilon_0 r$ that does not involve the equi-phase plane movement in the r direction, but changes the phase in time is a promising candidate.

The electric energy U_e' in the interval $[a,\infty]$ and the current energy U_i' in the interval $[0,b]$ are expressed as

$$U_e' = \frac{q_e^2}{32\pi^2\varepsilon_0}\int_a^\infty\left(\frac{1}{r^4}\right)4\pi r^2 dr = \frac{q_e^2}{8\pi\varepsilon_0}\cdot\frac{1}{a} \tag{16}$$

$$U_i' = \frac{q_e^2}{32\pi^2\varepsilon_0}\int_0^b\left(\frac{k^2}{r^4}\right)4\pi r^2 dr = \frac{q_e^2}{8\pi\varepsilon_0}k^2 b \tag{17}$$

Equations (16) and (17) present the maximum values of the electrical and current energies, respectively. When $k^2ab = 1$ satisfies the resonance condition in Eq. (6), the two are equal, and their sum is equal to the total energy U' in Eq. (14).

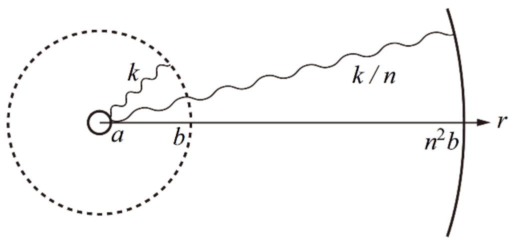

Figure 4. Outer radius and wave number after transition. n is the natural number. When the wave number changes from k to k/n, the outer radius changes from b to n^2b.

3.5 Resonance conditions and state transition

Consider the case where the basic parameters of a spherical standing wave, wave number k, inner radius a, and outer radius b change due to transition. After transition, let k' be the wave number, a' the inner radius, and b' the outer radius. The resonance condition after transition is maintained:

$$k'^2a'b' = 1 \tag{18}$$

where n is a natural number and wave number k' is the case where the original wave number k is divided by n. Assuming that the inner radius $a' = a$ remains unchanged, the outer radius after transition $b' = n^2b$ is given by Eqs. (6) and (18) [12]. Figure 4 shows the relationship between the outer radius and wave number after the transition. This transition is called the n state.

The electrical energy $U_e(n)$ and current energy $U_i(n)$ in section $[a, n^2b]$ transitioning to the n state are as

$$U_e(n) = \frac{q_e^2}{32\pi^2\varepsilon_0}\int_a^{n^2 b}\left(\frac{1}{r^4}\right)4\pi r^2 dr = \frac{q_e^2}{8\pi\varepsilon_0}\left(\frac{1}{a} - \frac{1}{n^2 b}\right) \tag{19}$$

$$U_i(n) = \frac{q_e^2}{32\pi^2\varepsilon_0}\int_a^{n^2 b}\left(\frac{k^2}{r^4}\right)4\pi r^2 dr = \frac{q_e^2}{8\pi\varepsilon_0}\cdot\frac{k^2}{n^2}(n^2 b - a) \tag{20}$$

In the n state, the wave range extends to $n^2 b$; however, the wave only exists in interval $[a, n^2 b]$; therefore, it is equivalent to the energy in the OFF state as in the case in Fig. 2.

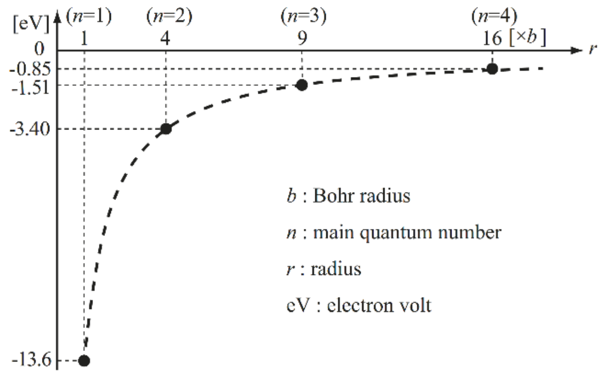

Figure 5. Principal quantum number dependence of ionization energy. The horizontal axis represents the distance from the center, while the vertical axis represents the energy in electron volts [eV].

3.6 Principal quantum number dependence of the ionization energy

Consider the ON state further after transition to n state. This state is considered to be ionization in n state. If the electric energy in this case is $U_e'(n)$, it is equal to the maximum value U_e' of the electric energy in Eq. (16) because the electric energy exists in section $[a, \infty]$.

The energy difference between the ON and OFF states in the n state is expressed as

$$U_e(n) - U_e' = -\frac{q_e^2}{8\pi\varepsilon_0}\cdot\frac{1}{n^2 b} \tag{21}$$

This equation demonstrates the principal quantum number dependence of the ionization energy of electrons. Moreover, Eq. (21) divided by q_e provides the ionization energy in electron volts. Fig. 5 shows the dependence of the ionization energy on the principal quantum number [13]. The energy takes a negative value because it is based on the maximum value of the electrical energy U_e'.

3.7 Derivation of the Rydberg constant

Let n and m be natural numbers (where $n > m$) and determine the difference in electrical energy when transitioning from the n state to the m state.

$$U_e(n) - U_e(m) = \frac{q_e^2}{8\pi\varepsilon_0 b}\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{22}$$

Think of this as the energy of the emitted light. If the angular frequency of light is ω_{nm} and the Planck constant is $\hbar = h/2\pi$, the photon energy equals $\hbar\omega_{nm}$. Using the relationship of $\omega_{nm} = 2\pi c/\lambda_{nm}$ and expressing it as the reciprocal of the wavelength, λ_{nm},

$$\frac{1}{\lambda_{nm}} = R_\infty\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \qquad R_\infty = \frac{q_e^2}{16\pi^2\varepsilon_0 b\hbar c} \tag{23}$$

and an equation representing the so-called spectral series of hydrogen atoms is derived. Here, R_∞ represents the Rydberg constant (Bohr radius is represented by b) [10]. Additionally, using the fine structure constant, $\alpha = q_e^2/4\pi\varepsilon_0\hbar c$,

$$R_\infty = \frac{\alpha}{4\pi b} \tag{24}$$

can be expressed. Moreover, the light is absorbed if $n < m$.

4. Conclusions

Table 1 shows the correspondence with the Bohr model [14]. Table 2 shows the calculation formulas and values of the model in this study. The values were calculated to 11 significant figures using the 2022 CODATA recommended values [15]. All the digits matched the existing physical constants.

Table 1. Comparison between the Bohr and spherical standing wave models

Electron (Bohr model)	Spherical standing waves
Elementary charge	Wave source size
Bohr radius	Outer radius
Classical electron radius	Inner radius
Compton wavelength	Wavelength
Mass	Total energy / c^2
Quantum condition	Resonance condition
Spin (up/down)	(Outward/inward) spherical wave

Table 2. Expression and calculated values using the spherical standing wave model
a: classical electron radius; b: Bohr radius; α: fine structure constant; q_e: elementary charge.

	Expression	Calculated values
Wave number	$k = 1/\sqrt{ab} = 1/\alpha b$	$2.589\ 605\ 0783 \times 10^{12}\ \mathrm{m}^{-1}$
(Compton) Wavelength	$\lambda = 2\pi\sqrt{ab} = 2\pi\alpha b$	$2.426\ 310\ 2354 \times 10^{-12}\ \mathrm{m}$
Mass (ground)	$m = \frac{\mu_0}{4\pi} q_e^2 k^2 b(1-\alpha^2)$	$9.108\ 898\ 6269 \times 10^{-31}\ \mathrm{kg}$
Mass (excited)	$m' = \frac{\mu_0}{4\pi} q_e^2 k^2 b$	$9.109\ 383\ 7139 \times 10^{-31}\ \mathrm{kg}$
Ionization energy	$-\frac{q_e}{8\pi\varepsilon_0} \cdot \frac{1}{n^2 b}$	$-13.605\ 693\ 123\ n^{-2}\ \mathrm{eV}$
Rydberg constant	$R_\infty = \frac{\alpha}{4\pi b}$	$1.097\ 373\ 1568 \times 10^{7}\ \mathrm{m}^{-1}$

Bohr's atomic model introduced a special assumption of quantum conditions instead of explaining why circularly moving electrons do not emit electromagnetic waves. Conversely, no special assumptions are necessary for the spherical standing wave model, wherein the range of existence of the wave is determined through the resonance conditions.

The mass of an electron can be accurately calculated, and this mass is a physical quantity that changes depending on the energy state. The self-energy divergence problem of point electrons has been solved without using renormalization [16].

Herein, we presented two types of excited states. The ionization energy is generated by the energy difference between the ON and OFF states without a wave number change. The light energy generates the difference through the transition states with a wave number change. Ion and light energies are calculated as the differences in the electrical energy between the states. However, simultaneously, the same amount of change in the current energy occurs, indicating the presence of unobserved energy.

The proposed model is compatible with quantum theory and relativity. Therefore, we expect to develop a theory that connects micro [17-19] and macro [6, 19].

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Data Availability Statement: The data that support the findings of this study are available within the article.

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